

JULY 9, 1973

NO. 80-001

DIRECT REDUCTION LOAN AMORTIZATION CALCULATIONS

General

The topics within this NOTE should be of assistance in using the HP-80 to solve problems in the category of *payment-to-principal/payment-to-interest* loans (e.g. mortgages).

First, the precise method of determining *accumulated interest/remaining balance* is shown. It is a result of, and a response to questions arising when an HP-80 solution has not compared exactly with that generated by other methods.

The remainder of the material concerns procedures for finding the *time to reach a specified balance*, accumulated interest when the current payment number is unknown, and generating an *amortization schedule*. Finally, the accumulated interest/remaining balance *equations* used by the HP-80 are included.

Using the HP-80 Accumulated Interest/Remaining Balance Calculation

The HP-80 assumes that the periodic payment amount entered is the *exact* payment to fully amortize the loan over its life. In many loans the payment amount has been rounded up to the nearest cent or dollar. To get the correct answer when the payment has been rounded, solve first for the exact number of time periods to fully amortize the loan and use this value in the accumulated interest/remaining principal calculation.

EXAMPLE:

What is the accumulated interest and remaining balance on a 25 year (300 month) 7% mortgage on \$20,000 where the monthly payments are \$141.40?

(The exact payment to amortize this loan over 300 months is \$141.355842)

PROCEDURE:

1. Solve for number of time periods to exactly pay off the loan and write this number down (we recommend writing it down to 6 decimal places).

Use these keystrokes:

7 **SAVE** 12 **÷** **i** 141.40 **PMT** 20000 **PV** **n** \longrightarrow 299.746473

(This is the exact number of months . . . write it down)

2. Now solve for accumulated interest and remaining balance. Use these keystrokes:

0 **STO** 12 **n** 299.746473 **n** 7 **SAVE** 12 **÷** **i** 141.40 **PMT** **Σ+** \longrightarrow 1390.29

(accumulated interest of \$1390.29)

x²y \longrightarrow 19693.49

(remaining balance of \$19693.49)

NOTE:

To get the accumulated interest for the first year (months 1–12) we begin the calculation by entering:

0 **STO** 12 **n** ...

To get the answers for the second year (months 13–24) we enter:

12 **STO** 24 **n** ...

The first time period that we enter is always *one less* than the actual first month number in the time span.

Finding Accumulated Interest for a Prior Period

We would like to thank Mr. Jacob Heskes of New York, New York, for the following HP-80 application:

Occasionally the payment number may not be known, but the remaining balance, interest rate and payments are known. It is still possible to find the accumulated interest for some prior period (i.e., the previous 12 months). Now, however, the month of the remaining balance is considered the first or reference period, and prior months are counted backwards from this point and have negative values.

EXAMPLE:

Assume a 7.5% loan that now has a remaining balance of \$1,367.04. Payments are \$118.71 per month. The payment number is unknown. How much interest has been paid over the past 12 months?

PROCEDURE:

1. First find (again to 6 decimal places) how many months it will take to pay off the remaining balance:

7.5 **SAVE** 12 **÷** **i** 118.71 **PMT** 1367.04 **PV** **n** → 11.988545

(This answer should be written down)

2. Now find accumulated interest for the last 12 months (time periods –12 to 0):

12 **CHS** **STO** 0 **n** 11.988545 **n** 7.5 **SAVE** 12 **÷** **i** 118.71 **PMT** **Σ+** → 154.70

(accumulated interest of \$154.70)

x↔y → 1367.04

(remaining balance of 1367.04 previously used)

Finding the Time to Reach a Specified Balance

The number of payment periods to reach a specified balance can be determined by first calculating the number of periods to pay off the remaining balance and subtracting that answer from the total payment periods.

EXAMPLE:

Assume a 30 year mortgage at 6.9% on \$45,854 with payments of \$301.99. How long will it take to reach a remaining balance of \$45,000?

PROCEDURE:

1. Calculate the time to pay off \$45,000

6.9 **SAVE** 12 **÷** **i** 301.99 **PMT** 45000 **PV** **n** → 338.99

2. Subtract from 360 months.

360 **x↔y** **-** → 21.01

(answer rounded is 21 months)

NOTE:

This is also a solution to the question, "How long will it take to build an equity of \$854?".

Direct Reduction Loan Amortization Schedule

This calculation generates the interest paid per period, the payment toward principal each period, and the remaining balance each period over the life of a direct reduction loan.

EXAMPLE:

Generate an amortization schedule for a loan with a principal amount of \$30,000, monthly payments of \$200, and an interest rate of 7%.

PROCEDURE:

1. Store payment amount and load monthly interest rate and beginning principal in the operational stack locations.

200 [CHS] [STO] 7 [SAVE ↑] 12 [÷] [SAVE ↑] [SAVE ↑] 30000

2. Calculate monthly payment on interest

[xzy] [%] → 175.00
(payment to interest on first payment)

3. Calculate payment toward principal

[RCL] [+] [CHS] → 25.00
(payment toward principal on first payment)

4. Calculate remaining balance

[-] → 29975.00
(remaining balance after first payment)

5. Return to step number 2 to calculate values for next time period.

HP-80 Accumulated Interest/Remaining Balance Equations

The equations used by the HP-80 are:

$$I_{j-k} = PMT \left[k - j - \frac{(1+i)^{k-n}}{i} + \frac{(1+i)^{j-n}}{i} \right]$$

$$PV_k = \frac{PMT}{i} \left[1 - (1+i)^{k-n} \right]$$

WHERE:

I_{j-k} = accumulated interest from payment j to payment k

PV_k = remaining balance after payment k

PMT = period payment amount

i = periodic interest rate

n = total number of payment periods

JULY 30, 1973

NO. 80-002

ANNUITY DUE CALCULATIONS FOR SAVINGS FUNDS

GENERAL

Data entry sequences for the HP-80 top row keys (**n** **i** **PMT** **PV** **FV**) were designed to be simplest for calculations involving ordinary annuities. The most common definition of *ordinary annuity* (sometimes called “payments in arrears”) assumes that payment amounts are equal, that payment periods are equally spaced in time, and that these payments are made *at the end* of each period. However, by slightly modifying the standard key sequences, the HP-80 can be used for *annuity due* calculations (sometimes called “payments in advance”) where equal payments are made *at the beginning* of equally spaced periods.

This issue of the HP-80 APPLICATION NOTES will be of assistance to the HP-80 user wishing to solve for *Future value, payment amount, number of payment periods, and interest rate* in annuity due calculations.

The symbolic values listed below will be used to demonstrate the various keystroke sequences that follow.

A = number of payment periods in a year

B = number of years

C = annual interest rate

D = payment amount

F = future value of the series of payments at the end of the last payment period

FIND THE FUTURE VALUE OF A SERIES OF PAYMENTS


Keystrokes:

1. A **STO** **SAVE ↑** B **×** **n**
2. C **RCL** **÷** **STO** **i**
3. D **RCL** **%** **+** **PMT**
4. **FV**  F

Example:

Deposits of \$75 a month for 6 years will be paid into a savings account starting tomorrow. The savings plan compounds monthly using a 5% nominal interest rate. How much money will be in the account at the end of 6 years?

Procedure:

12 **STO** **SAVE ↑** 6 **×** **n** 5 **RCL** **÷** **STO** **i** 75 **RCL** **%** **+** **PMT** **FV**  6308.494583

(This series of payments will be worth \$6,308.49)

Note:

When solved using ordinary annuity assumptions the future value of these payments would be \$6,282.32.

FIND THE PAYMENT AMOUNT REQUIRED TO ACHIEVE SOME FUTURE VALUE

Keystrokes:

1. A **STO** **SAVE ↑** B **×** **n**
2. C **RCL** **÷** **STO** **i**
3. 1 **RCL** **%** **+** F **xzy** **÷** **FV**
4. **PMT** \longrightarrow D

Example:

It has been determined that \$10,000 will be required 15 years from now to send a son to college. A savings plan paying 7% (compounded monthly) is available. What equal monthly payments should be made in order to accrue the required amount?

Procedure:

- 12 **STO** **SAVE ↑** 15 **×** **n** 7 **RCL** **÷** **STO** **i** 1 **RCL** **%**
- +** 10000 **xzy** **÷** **FV** **PMT** \longrightarrow 31.366525
(\$31.37 per month)

FIND THE NUMBER OF PERIODS WHEN **PMT **FV** AND **i** ARE KNOWN**

Keystrokes:

1. C **SAVE ↑** A **÷** **STO** **i**
2. D **RCL** **%** **+** **PMT**
3. F **FV**
4. **n** \longrightarrow AxB

Example:

The same \$10,000 amount is desired as in the previous example and a 7% interest also applies. Now however, the individual will be placing \$40 per month in his account. How long will it be before the desired sum is available?

Procedure:

- 7 **SAVE ↑** 12 **÷** **STO** **i** 40 **RCL** **%** **+** **PMT** 10000 **FV** **n** \longrightarrow 154.053987
(154 months)
- 12 **÷** \longrightarrow 12.837832
(almost 13 years)

FIND THE ANNUAL INTEREST RATE

Keystrokes:

1. A **SAVE ↑** B **×** 1 **+** **n**
2. D **SAVE ↑** **PMT**
3. F **+** **FV**
4. **i** A **×** \longrightarrow C

Example:

Suppose the individual in our second example was only able to pay \$20 per month but still wanted to accumulate \$10,000 in 15 years. What interest rate must a savings plan offer in order to accomplish this?

Procedure:

- 12 **SAVE ↑** 15 **×** 1 **+** **n** 20 **SAVE ↑** **PMT** 10000 **+** **FV** **i** 12 **×** \longrightarrow 11.904768
(an 11.90% interest rate is required)

AUGUST 17, 1973

NO. 80-003

ANNUITY DUE CALCULATIONS FOR SAVINGS PLANS WHEN COMPOUNDING PERIODS DIFFER FROM PAYMENT PERIODS

GENERAL

In financial calculations involving a series of payments equally spaced in time with periodic compounding, both periods of time are normally equal and coincident. This is an assumption built into the HP-80 pre-programming.

In savings plans however, money may become available for deposit or investment at a frequency different from the compounding frequencies offered. The HP-80 can easily be used in these calculations. However, because of the assumptions mentioned, adjustments must be made to the data so that the two periods can be considered to occur at the same time. When the compounding periods occur more frequently than payment periods, additional keystrokes adjust the compounding period interest rate to an equivalent rate for the payment period. When payments occur more frequently than compounding, the payment amount is adjusted to reflect the fact that payments accrue simple interest between compounding periods.

This issue presents keystroke solutions for *future value, payment amount, and number of payments* for both situations of compounding periods differing from payment periods. In addition it should be noted that only annuity due (payments at the beginning of payment periods) calculations are shown since this is most common in savings plan calculations.

The following symbolic values will be used in the keystroke sequences shown below:

- A = number of payment periods in a year
- U = number of compounding periods in a year
- B = number of years (and/or fraction of a year as appropriate)
- C = annual interest rate expressed as a percent
- D = payment amount
- F = future value of the series of payments at the end of the last payment period

COMPOUNDING PERIODS MORE FREQUENT

Keystrokes (solving for future value):

- Calculate an equivalent payment period interest rate and store it.

U **STO** **SAVE** **A** **÷** **n** **C** **RCL** **÷** **i** **100** **STO** **PV** **FV** **RCL** **-** **STO**

- Assuming payments are made at the beginning of the payment periods, calculate the future value.

A **SAVE** **B** **x** **n** **RCL** **i** **D** **RCL** **%** **+** **PMT** **FV** **→** F

Example:

Quarterly deposits (A = 4) of \$95 (D) are to be made into a savings account paying 5% (C), compounded monthly (U = 12). What amount (F) will be in the account at the end of 7 (B) years?

Procedure:

1. Calculate the equivalent interest rate

12 [STO] [SAVE ↑] 4 [÷] [n] 5 [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [-] [STO] → 1.255215

2. Calculate the future value

4 [SAVE ↑] 7 [x] [n] [RCL] [i] 95 [RCL] [%] [÷] [PMT] [FV] → 3203.586230
(\$3203.59)

Keystrokes (solving for payment amount):

1. Calculate an equivalent interest rate

U [STO] [SAVE ↑] A [÷] [n] C [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [-] [STO]

2. Calculate the payment amount assuming again that payments occur at the beginning of payment periods.

A [SAVE ↑] B [x] [n] [RCL] [i] 1 [RCL] [%] [÷] F [x]y [÷] [FV] [PMT] → D

Example:

A sum of money will become available annually (A = 1) for 20 years (B) starting tomorrow. It is desired to place this money in a savings institution paying 7% (C) compounded daily (U = 365, see NOTE below) so that at the end of 20 years \$19,000 (F) will have been accumulated. How large must this annual deposit be?

Procedure:

1. Calculate the equivalent interest

365 [STO] [SAVE ↑] 1 [÷] [n] 7 [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [-] [STO] → 7.250066

2. Calculate the annual payment

1 [SAVE ↑] 20 [x] [n] [RCL] [i] 1 [RCL] [%] [÷] 19000 [x]y [÷] [FV] [PMT] → 420.474071
(\$420.47)

Note: Using 365 or 360 (days) as the number of compounding periods in a year when daily compounding is available would depend on the practice of the particular institution being considered.

Keystrokes (solving for number of payment periods)

1. Calculate the equivalent interest rate

U [STO] [SAVE ↑] A [÷] [n] C [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [-] [STO]

2. Calculate number of payments

[i] D [RCL] [%] [÷] [PMT] F [FV] [n] → A · B
(total number of payments)

Example:

Semi-annual payments (A = 2) of \$1200 (D) are to be made into an account paying 8% (C) compounded monthly (U = 12). How many payment periods will it take to accrue \$25,000?

Procedure:

1. Calculate the equivalent interest rate

12 [STO] [SAVE ↑] 2 [÷] [n] 8 [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [-] [STO] → 4.067262

2. Now calculate the number of payments

[i] 1200 [RCL] [%] [÷] [PMT] 25000 [FV] [n] → 14.941105
(approximately 15 semi annual payments)

PAYMENT PERIODS MORE FREQUENT

Keystrokes (solving for future value):

- Assuming payments occur at the beginning of payment periods, calculate an equivalent *payment amount* that can be considered to occur with the same frequency as compounding.

A U 1 2 C D

- Using this equivalent payment amount, future value can be calculated.

U B C U \longrightarrow F

Example:

Deposits of \$100 (D) per month (A = 12) will be made into a savings account paying 6% (C) compounded quarterly (U = 4).

Find the value of the series of payments at the end of 14 (B) years.

Procedure:

- Calculate an equivalent quarterly payment.

12 4 1 2 6 100 \longrightarrow 303.00

- Now calculate the future value.

4 14 6 4 \longrightarrow 26299.65551
(\$26,299.66)

Keystrokes (solving for payment):

Unlike the other situations presented, in solving for payment amount when payments occur more frequently than compounding, the adjustments are made to an HP-80 answer rather than to input data.

- Calculate and store a payment factor to be recalled for use in step 3.

A U 1 2 C

- Using the ordinary annuity keystrokes solve for the intermediate payment amount if payments were to occur with the same frequency as compounding.

U B C U F

- Now divide by the payment factor calculated in step 1.

\longrightarrow D

Example:

A mutual fund has consistently paid annual dividends (U = 1) of 6% (C) on money invested, and offers investors the opportunity to automatically reinvest this money. In addition, investors can make monthly (A = 12) investments in the fund. Assuming the dividend rate stays constant and without considering stock appreciation, what should the monthly payments be if an investor wishes to accrue an investment of \$25,000 (F) at the end of 15 years (B)?

Procedure:

- Calculate and store the monthly payment factor

12 1 1 2 6 \longrightarrow 12.39

- Calculate the equivalent annual payment to reach \$25,000 at the end of 15 years at 6%.

1 15 6 1 25000 \longrightarrow 1074.06910

3. Divide by the payment factor to find the monthly payment required.

$$\boxed{\text{RCL}} \boxed{\div} \longrightarrow 86.688386$$

$$(\$86.69)$$

Keystrokes (solving for number of payment periods):

1. Calculate and store the equivalent payment amount.

$$A \boxed{\text{SAVE}} \boxed{\uparrow} \boxed{\text{SAVE}} \boxed{\uparrow} U \boxed{\text{STO}} \boxed{\div} 1 \boxed{+} 2 \boxed{\div} C \boxed{\%} \boxed{\text{x}\div\text{y}} \boxed{\text{R}\div} \boxed{+} \boxed{\text{RCL}} \boxed{\div} D \boxed{\times} \boxed{\text{STO}}$$

2. The number of payment periods is arrived at by first calculating the number of compounding periods.

$$C \boxed{\text{SAVE}} \boxed{\uparrow} U \boxed{\div} \boxed{i} \boxed{\text{RCL}} \boxed{\text{PMT}} \boxed{\text{F}} \boxed{\text{FV}} \boxed{n} \longrightarrow U \cdot B$$

(Total compounding periods)

3. Dividing by the compounding periods per year results in total years. . .

$$U \boxed{\div} \longrightarrow B$$

4. and multiplying by payment periods per year gives the number of payments required.

$$A \boxed{\times} \longrightarrow A \cdot B$$

(Total payment periods)

Example:

Deposits of \$100 (D) per month (A = 12) will be made into a savings account paying 6% (C) compounded quarterly (U = 4). How long (B), and how many payments will it take to accrue \$26,299.66?

Procedure:

1. Calculate and store an equivalent payment amount.

$$12 \boxed{\text{SAVE}} \boxed{\uparrow} \boxed{\text{SAVE}} \boxed{\uparrow} 4 \boxed{\text{STO}} \boxed{\div} 1 \boxed{+} 2 \boxed{\div} 6 \boxed{\%} \boxed{\text{x}\div\text{y}} \boxed{\text{R}\div} \boxed{+}$$

$$\boxed{\text{RCL}} \boxed{\div} 100 \boxed{\times} \boxed{\text{STO}} \longrightarrow 303.00$$

2. Find the total compounding periods.

$$6 \boxed{\text{SAVE}} \boxed{\uparrow} 4 \boxed{\div} \boxed{i} \boxed{\text{RCL}} \boxed{\text{PMT}} 26299.66 \boxed{\text{FV}} \boxed{n} \longrightarrow 56.000006$$

3. Find the number of years.

$$4 \boxed{\div} \longrightarrow 14.000002$$

(14 years)

4. Calculate total number of payments.

$$12 \boxed{\times} \longrightarrow 168.000019$$

(168 payments)

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NO. 80-004

ANNUAL PERCENTAGE RATE (APR) FOR LOANS WITH A BALLOON PAYMENT; RATE OF RETURN FOR EVEN CASH FLOWS TERMINATED BY AN UNEVEN CASH FLOW

GENERAL

This note will be useful to the HP-80 user wishing to calculate the *Annual Percentage Rate (APR)*, and sometimes called the actuarial rate) of loans terminated by a balloon payment, or the rate of return (often called yield) of a series of even cash flows terminated by an uneven cash flow.

The following symbolic values are used in demonstrating the general keystroke solutions.

- A = number of payment periods in a year
- B = number of years (and/or fraction of a year as appropriate)
- C = Annual Percentage Rate (APR), or rate of return expressed as a percent
- D = periodic payment amount (cash flow)
- E = initial principal amount of loan (initial investment)
- G = balloon payment amount or remaining balance

ANNUAL PERCENTAGE RATE FOR LOANS WITH A BALLOON PAYMENT

When the initial principal (E), total number of payments (A × B), payment amount (D) and balloon payment (G) amount are known, the HP-80 can be used to solve for the Annual Percentage Rate (APR) of the loan. These keystrokes are valid regardless of the way in which the loan was initially constructed (i.e. direct reduction, add-on rate etc.)

Keystrokes (when the balloon payment occurs in the same time period as the last payment)

1. Divide the balloon by 100 and store the result.

G **SAVE** 100 **÷** **STO**

2. Calculate the number of payments, multiply by 365, divide by 2 and press **n**.

A **SAVE** B **×** 365 **×** 2 **÷** **n**

3. Multiply the payment amount by 2, divide by the number stored in step 1. and press **PMT**.

D **SAVE** 2 **×** **RCL** **÷** **PMT**

4. Divide the loan amount by the number stored in step 1. and press **PV**.

E **RCL** **÷** **PV**

5. Calculate the APR.

▨ (gold key) **i** A **×** 2 **÷** **→** C

Example:

Find the annual interest rate (APR) on a \$2100 (E) loan requiring equal monthly (A = 12) payments of \$42.52 (D) for 2 years (B) and a balloon payment of \$1500 (G) due in addition to the final payment at month 24.

Procedure:	See Displayed:
1. 1500 SAVE ↑ 100 ÷ STO →	15.00
2. 12 SAVE ↑ 2 × 365 × 2 ÷ n →	4380.00
3. 42.52 SAVE ↑ 2 × RCL ÷ PMT →	5.67
4. 2100 RCL ÷ PV →	140.00
5. ▨ (gold key) i 12 × 2 ÷ →	11.53% (APR)

Keystrokes: (when the balloon payment occurs 1 period after the last payment).

- Subtract one payment from the balloon payment, divide by 100 and store the result.
G **SAVE ↑** D **-** 100 **÷** **STO**
- Calculate the total number of payments, including the balloon payment, multiply by 365, divide by 2 and press **n**.
A **SAVE ↑** B **×** 1 **+** 365 **×** 2 **÷** **n**
- Enter payment amount, multiply by 2, divide by the number stored in step 1. and press **PMT**.
D **SAVE ↑** 2 **×** **RCL** **÷** **PMT**
- Enter the loan amount, divide by the number stored in step 1. and press **PV**.
E **RCL** **÷** **PV**
- Solve for the APR.
▨ (gold key) **i** A **×** 2 **÷** → C

Example:

Assuming that the balloon payment of \$1500 in the previous example was due one month after the last payment (i.e. at month 25), and all other factors remained the same, what would the APR be?

Procedure:	See Displayed:
1. 1500 SAVE ↑ 42.52 - 100 ÷ STO →	14.57
2. 12 SAVE ↑ 2 × 1 + 365 × 2 ÷ n →	4562.50
3. 42.52 SAVE ↑ 2 × RCL ÷ PMT →	5.83
4. 2100 RCL ÷ PV →	144.08
5. ▨ (gold key) i 12 × 2 ÷ →	11.19% (APR)

RATE OF RETURN

The keystrokes below calculate the rate of return for situations that involve some initial payment followed by even cash flows (equal amounts, equally spaced in time) and a final uneven cash flow (balloon payment) coincident with the last payment.

Keystrokes:

The keystrokes shown here are exactly the same as those shown for calculating the APR of a loan with a balloon payment when the balloon payment occurs at the same time as the last periodic payment.

1. G $\boxed{\text{SAVE} \uparrow}$ 100 $\boxed{\div}$ $\boxed{\text{STO}}$
2. A $\boxed{\text{SAVE} \uparrow}$ B $\boxed{\times}$ 365 $\boxed{\times}$ 2 $\boxed{\div}$ \boxed{n}
3. D $\boxed{\text{SAVE} \uparrow}$ 2 $\boxed{\times}$ $\boxed{\text{RCL}}$ $\boxed{\div}$ $\boxed{\text{PMT}}$
4. E $\boxed{\text{RCL}}$ $\boxed{\div}$ $\boxed{\text{PV}}$
5. $\boxed{\text{gold key}}$ (gold key) \boxed{i} A $\boxed{\times}$ 2 $\boxed{\div}$ \longrightarrow C

Example:

Assume that something (property, rights to a franchise, etc.) is purchased for \$200,000 (E), and that it generates an annual income (A = 1) of \$12,000 (D) for 6 (B) years. If it is sold at the end of the 6 years for \$300,000 (G), what is the annual rate of return?

Procedure:

Procedure:	See Displayed:
1. 300000 $\boxed{\text{SAVE} \uparrow}$ 100 $\boxed{\div}$ $\boxed{\text{STO}}$ \longrightarrow	3000.00
2. 1 $\boxed{\text{SAVE} \uparrow}$ 6 $\boxed{\times}$ 365 $\boxed{\times}$ 2 $\boxed{\div}$ \boxed{n} \longrightarrow	1095.00
3. 12000 $\boxed{\text{SAVE} \uparrow}$ 2 $\boxed{\times}$ $\boxed{\text{RCL}}$ $\boxed{\div}$ $\boxed{\text{PMT}}$ \longrightarrow	8.00
4. 200000 $\boxed{\text{RCL}}$ $\boxed{\div}$ $\boxed{\text{PV}}$ \longrightarrow	66.67
5. $\boxed{\text{gold key}}$ (gold key) \boxed{i} 1 $\boxed{\times}$ 2 $\boxed{\div}$ \longrightarrow	12.14% (rate of return)

Notes:

1. When calculating the rate of return, if the periodic payments (D) represent a cash outflow instead of an income this value would be entered as a negative number.
2. The HP-80 yield to maturity calculation for bonds is used to solve the problems in this issue. Solving for APR or rate of return is analogous to solving for bond yield. The property price or initial loan amount corresponds to the bond price. Income from property or loan payments correspond to bond coupons, and the balloon payment on a loan, or sale price in the rate of return calculation compares to the redemption (face) value of a bond.

The HP-80 bond calculations have built-in assumptions which are specifically tied to bond calculations. Some of these are:

- Bond coupons are paid semiannually
- Time is entered in days
- Bond price is expressed as a percent of redemption value

To change these assumptions and use another set of conditions, the data must be adjusted. This is the reason for all the numerical data entries (i.e., 365 $\boxed{\times}$, 2 $\boxed{\div}$) and the $\boxed{\text{RCL}}$ $\boxed{\div}$ sequence in the general symbolic keystroke solutions shown.

3. Since the bond yield algorithm is being used, the same operating limits as expressed in Appendix D of the *HP-80 Owner's Handbook* would apply. For these applications these limits can be expressed as follows:
The absolute value of the number entered for $\boxed{\text{PMT}}$ must be greater than .125 and less than the value entered for $\boxed{\text{PV}}$. The value entered for $\boxed{\text{PV}}$ must be greater than 20 and less than 5000.
4. It should be noted that for the calculations in this note the periodic payments (D) are assumed to occur *at the end* of the payment periods (ordinary annuity).

HP-80

APPLICATION NOTES

PUBLISHED AS A SERVICE FOR USERS OF THE HP-80 FINANCIAL POCKET CALCULATOR

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PRICE AND YIELD CALCULATIONS FOR MORTGAGES TRADED AT A DISCOUNT/PREMIUM

GENERAL

This note will be useful to HP-80 users who buy and/or sell mortgages at prices lower (discounted) or higher (at a premium) than the remaining balance of the loan at the time of purchase.

Keystrokes solving for *annual yield* (given a price) and *price* (given a desired yield) are shown for mortgages both *with and without balloon payments*.


Most mortgage loans are not paid to maturity, but are prepaid, and values for yield or price change depending on the time of prepayment. While there are regional assumptions available for estimating the time of prepayment is often desirable to determine values for price or yield for several prepayment assumptions. This necessitates generating a remaining balance (balloon payment) for each new assumption, and for this reason a simplified method for *calculating the remaining balance* of a direct reduction loan (mortgage loan) has been included.

The following symbolic values will be used in the keystroke sequences shown below:

- A = Number of payment periods in a year
- B = Number of years (and/or fraction of a year as appropriate).
- C = Annual yield (for price and yield calculations) or annual percentage rate (for remaining balance calculations).
- D = Periodic payment amount
- E = Purchase price (for yield and price calculations), or mortgage amount (for remaining balance calculations).
- G = Balloon payment amount, remaining balance.

MORTGAGES WITH A BALLOON PAYMENT

The balloon payment of a mortgage may occur in one of two ways. It may be part of the original loan. That is, it is agreed that the mortgage is to be repaid by a series of equal payments plus a balloon payment. Alternatively, the schedule of payments is arranged such that the loan is to be fully amortized (without a balloon) by a series of equal payments, but the mortgage is paid in full prior to maturity. The balloon payment in this case would at least be equal to the remaining balance of the mortgage at the time of prepayment. (The balloon payment could also include prepayment penalties, etc.). The keystroke solutions shown are valid for either situation. However, the examples refer to balloon payments on prepaid mortgages, since this situation occurs more frequently. It should be noted that the balloon payment amount, for the purposes of these keystrokes, occurs coincident with, and does not include the last periodic payment amount.

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Keystrokes (yield):

1. Divide the balloon payment that will occur at the time of prepayment by 100 and store the result.
G **SAVE ↑** 100 **÷** **STO**
2. Enter the number of payments from the time of purchase to the time of prepayment, divide by 2, multiply by 365 and press **n** .
A **SAVE ↑** B **×** 2 **÷** 365 **×** **n**
3. Enter the periodic payment amount, multiply by 2, recall and divide by the value stored in step 1., and press **PMT** .
D **SAVE ↑** 2 **×** **RCL** **÷** **PMT**
4. Enter purchase amount, recall and divide by the value stored in step 1., and press **PV** .
E **RCL** **÷** **PV**
5. Calculate the annual yield.
▨ (gold key) **i** A **×** 2 **÷** \longrightarrow C

Example:

A mortgage with monthly (A=12) payments of \$265.07 (D) can be purchased for \$35,000 (E). It is assumed that this mortgage will be prepaid in 7 years (B), and there will be a remaining balance (balloon payment) of \$34,099.87 (G) at that time. Calculate the annual yield (C) if these conditions prevail.

Procedure:

	<i>See Displayed</i>
1. 34099.87 SAVE ↑ 100 ÷ STO \longrightarrow	341.00
2. 12 SAVE ↑ 7 × 2 ÷ 365 × n \longrightarrow	15330.00
3. 265.07 SAVE ↑ 2 × RCL ÷ PMT \longrightarrow	1.55
4. 35000 RCL ÷ PV \longrightarrow	102.64
5. ▨ (gold key) i 12 × 2 ÷ \longrightarrow	8.82 (8.82% annual yield)

Notes:

1. The HP-80 yield to maturity programming for bonds can be used for finding the yield because the cash flows of bond and mortgage transactions are analogous. The price of a mortgage corresponds to the price of a bond, the periodic mortgage payments correspond to bond coupons, and the balloon payment of a prepaid mortgage compares to the redemption (face) value of a bond.

The HP-80 bond yield programming has built-in assumptions, however, which are specifically tied to bond calculations.

Some of these are:

- Bond coupons are paid semiannually
- Time is entered in days
- Bond price is expressed as a percent of redemption value

To change these assumptions and use another set of conditions, the data must be adjusted. This is the reason for all the numerical data entries (i.e., 365 **×** , 2 **÷**) and the **RCL** **÷** sequence in the general symbolic keystroke solutions shown.

2. Since the bond yield algorithm is being used, the same operating limits as expressed in Appendix D of the *HP-80 Owner's Handbook* apply. For these applications the limits can be expressed as follows:
*The absolute value of the number entered for **PMT** must be greater than .125 and less than the value entered for **PV**. The absolute value entered for **PV** must be greater than 20 and less than 5000.*

Keystrokes (price):

- Determine the present value of the periodic payments from the time of purchase to the time of prepayment using the desired yield. Store the result.
A **STO** **SAVE ↑** B **×** **n** C **RCL** **÷** **i** D **PMT** **PV** **STO**
- Using the desired yield, the balloon payment, and the same number of payment periods, find the present value of that balloon payment amount.
A **SAVE ↑** B **×** **n** C **SAVE ↑** A **÷** **i** G **FV** **PV**
- Find the price to pay by adding these two results.
RCL **+** $\xrightarrow{\hspace{15em}}$ E

Example:

A mortgage with monthly (A=12) payments of \$271.49 (D) can be purchased. It is assumed that the mortgage will be prepaid in 12 years (B), and the remaining balance (balloon payment amount) at that time will be \$31,029.08 (G). Determine the price to pay for this mortgage if the desired annual yield (C) is 13%.

Procedure:

- | | | |
|----|--|---|
| | | <i>See Displayed</i> |
| 1. | 12 STO SAVE ↑ 12 × n 13 RCL ÷ i 271.49 PMT PV STO $\xrightarrow{\hspace{1em}}$ | 19750.11
(present value of series of payments) |
| 2. | 12 SAVE ↑ 12 × n 13 SAVE ↑ 12 ÷ i 31029.08 FV PV $\xrightarrow{\hspace{1em}}$ | 6575.26
(present value of the remaining balance) |
| 3. | RCL + $\xrightarrow{\hspace{15em}}$ | 26325.37
(\$26,325.37, total present value) |

CALCULATING THE REMAINING BALANCE OF A DIRECT REDUCTION LOAN (MORTGAGE)

The HP-80 is programmed to calculate the remaining balance and accumulated interest at a particular point in time and this procedure is outlined in the "HP-80 Owner's Handbook". The keystrokes below are presented as an alternative if only the remaining balance is desired.

- Calculate the exact number of periods required to pay off the mortgage.
C **SAVE ↑** 12 **÷** **i** D **PMT** E **PV** **n**
- Subtract from the value just calculated the number of the payment period in which the remaining balance will occur and press **n**.
A **SAVE ↑** B **×** **-** **n**
- The remaining keystrokes are the same as those that calculate the present value of a series of equal payments.
C **SAVE ↑** 12 **÷** **i** D **PMT** **PV** $\xrightarrow{\hspace{15em}}$ G

Note:

Step 1 is required only if the exact remaining balance is desired, because payment amounts are often rounded to the nearest cent, ten cents, or dollar, with the last payment adjusting for this rounding. A close approximation of the remaining balance can be achieved by replacing the calculation shown for step 1 with a simple entry of the actual total of number of payments. This ignores the fact that the last payment is not equal to the rest, but the amount of error in the remaining balance calculated in this manner would not generally affect the calculated values for price or yield significantly.

Example:

A 30 year (360 payments) loan of \$20,000 (E), with an 8% (C) annual interest rate, is being paid back with 359 monthly (A=12) payments of \$146.75 (D) and a final payment (360) of \$151.10. What will be the outstanding balance (G) at the end of 7 (B) years.

Procedure:

- | | | <i>See Displayed</i> |
|----|---|--|
| 1. | 8 <input type="button" value="SAVE ↑"/> 12 <input type="button" value="÷"/> <input type="button" value="i"/> 146.75 <input type="button" value="PMT"/> 20000 <input type="button" value="PV"/> <input type="button" value="n"/> → | 360.03 |
| | (This is the exact number of \$146.75 payments required) | |
| 2. | 12 <input type="button" value="SAVE ↑"/> 7 <input type="button" value="×"/> <input type="button" value="-"/> <input type="button" value="n"/> → | 276.03 |
| 3. | 8 <input type="button" value="SAVE ↑"/> 12 <input type="button" value="÷"/> <input type="button" value="i"/> 146.75 <input type="button" value="PMT"/> <input type="button" value="PV"/> → | 18495.81
(\$18,495.81—remain-
ing balance) |

Example:

Calculate the remaining balance for the preceding example, replacing step 1 by simply entering the total number of payments (360).

Procedure:

- | | | <i>See Displayed</i> |
|----|--|--|
| 1. | 360 <input type="button" value="SAVE ↑"/> → | 360.00 |
| 2. | 12 <input type="button" value="SAVE ↑"/> 7 <input type="button" value="×"/> <input type="button" value="-"/> <input type="button" value="n"/> → | 276.00 |
| 3. | 8 <input type="button" value="SAVE ↑"/> 12 <input type="button" value="÷"/> <input type="button" value="i"/> 146.75 <input type="button" value="PMT"/> <input type="button" value="PV"/> → | 18495.12
(\$18,495.12—remain-
ing balance) |

MORTGAGES WITHOUT BALLOON PAYMENTS

If the schedule of payments will fully amortize a loan and it is not prepaid there will of course be no balloon payment. The keystrokes for yield and price in this case are simple top row calculations.

Keystrokes (yield):

A B D E A → C

Example:

A mortgage with monthly (A=12) payments of \$311.12 (D) has a life of 7 years (B). It is assumed the mortgage will be held for the full 7 years and the mortgage can be purchased for \$17,000 (E). Determine the yield.

Procedure:

12 **SAVE ↗** 7 **×** **n** 311.12 **PMT** 17000 **PV** **i** 12 **×** *See Displayed*
13.20
(13.20% – annual yield)

Keystrokes (price):

A **STO** **SAVE ↗** B **×** **n** C **RCL** **÷** **i** D **PMT** **PV** E

Example:

Assume that the payments (D=\$311.12) and the term (A=12 payments/year, B=7 years) of the preceding example remain the same, but a 14% (C) yield is desired. Determine the price to pay for this mortgage to achieve this yield.

Procedure:

12 **STO** **SAVE ↗** 7 **×** **n** 14 **RCL** **÷** **i** 311.12 **PMT** **PV** *See Displayed*
16601.91
(\$16,601.91)

November 16, 1973

NO. 80-006

ANNUITY DUE AND PRESENT VALUE

GENERAL

Data entry sequences for the HP-80 top row keys (**n** **i** **PMT** **PV** **FV**) were designed to be simplest for calculations involving ordinary annuities. The most common definition of *ordinary annuity* (sometimes called "payments in arrears") assumes that payment amounts are equal, that payment periods are equally spaced in time, and that these payments are made *at the end* of each period. However, by slightly modifying the standard key sequences, the HP-80 can be used for *annuity due* calculations (sometimes called "payments in advance") where equal payments are made *at the beginning* of equally spaced periods.

This issue of the HP-80 APPLICATION NOTES will be of assistance to the HP-80 user wishing to solve for *present value*, *payment amount*, number of *payment periods*, and *interest rate* in annuity due calculations.

The symbolic values listed below will be used to demonstrate the various keystroke sequences that follow.

- A = number of payment periods in a year
- B = number of years
- C = annual interest rate or annual yield (expressed as a %)
- D = payment amount
- E = present value of a series of payments

FIND THE PRESENT VALUE OF A SERIES OF PAYMENTS

Keystrokes:

1. A **STO** **SAVE** ↑ B **X** **n**
2. C **RCL** **÷** **STO** **i**
3. D **RCL** **%** **+** **PMT**
4. **PV** → E

Example:

The owner of a downtown parking lot has been able to achieve full occupancy and a 7% annual yield (C) by renting parking spaces for \$40 (D) per month (A = 12), payable in advance. Some regular customers have expressed interest in renting their spaces on an annual basis. What minimum annual (B = 1) rent, also payable in advance, will maintain his 7% annual yield rate?

Procedure:

- 12 **STO** **SAVE** ↑ 1 **X** **n** 7 **RCL** **÷** **STO** **i** 40 **RCL** **%** **+** **PMT** **PV** → 464.98
 (he should charge \$464.98 annually)

FIND THE PERIODIC PAYMENT AMOUNT

Keystrokes:

1. A **STO** **SAVE** B **X** **n**
2. C **RCL** **÷** **STO** **i**
3. 1 **RCL** **%** **+** E **xzy** **÷** **PV**
4. **PMT** → D

Example:

The owner of a piece of equipment presently worth \$70,000 (E) intends to lease the equipment for a 5 year period (B). He estimates that the equipment will have no salvage value at the end of the lease, and he desires a 7% annual yield (C). What should the quarterly payments (A = 4) be, assuming that payments are made at the beginning of each quarter?

Procedure:

- 4 **STO** **SAVE** 5 **X** **n** 7 **RCL** **÷** **STO** **i** 1 **RCL** **%** **+** 70000
- xzy** **÷** **PV** **PMT** → 4106.52
(\$4106.52 per quarter)

FIND THE NUMBER OF PERIODS

Keystrokes:

1. C **SAVE** A **÷** **STO** **i**
2. D **RCL** **%** **+** **PMT**
3. E **PV**
4. **n** → AxB

Example:

The owner of the equipment in the previous example feels that it would be better if the quarterly payments (A = 4) on the \$70,000 (E) piece of equipment were \$3,600 (D). How long would he have to lease the equipment if he still desires a 7% annual yield (C).

Procedure:

- 7 **SAVE** 4 **÷** **STO** **i** 3600 **RCL** **%** **+** **PMT** 70000 **PV** **n** → 23.47
(23.47 quarters)
- 4 **÷** → 5.87
(almost 6 years)

FIND THE ANNUAL INTEREST RATE OR YIELD

Keystrokes:

1. A **SAVE** B **X** 1 **-** **n**
2. D **SAVE** **PMT**
3. E **xzy** **-** **PV**
4. **i** A **X** → C

Example:

A term insurance policy may be paid in two ways. The policy holder may pay \$1000 (E) at the beginning of each year (B = 1) or he may elect to make payments of \$84.87 (D) at the beginning of each month (A = 12). What is the apparent annual interest rate the insurance company is using when converting from annual payments to monthly payments?

Procedure:

- 12 **SAVE** 1 **X** 1 **-** **n** 84.87 **SAVE** **PMT** 1000 **xzy** **-** **PV** **i** 12 **X** → 4.01
(4.01%)

DECEMBER 10, 1973

NO. 80-007

CALCULATING LOGS, ANTI-LOGS, AND ROOTS OF NUMBERS

GENERAL

This Application Note will be of assistance to those HP-80 owners who require the ability to calculate *common and natural logarithms, anti-logarithms (base 10 and base e), and the "nth" root of a number.*

COMMON (BASE 10) AND NATURAL (BASE e) LOGARITHMS

The following keystrokes will simultaneously calculate the common logarithm (log) and natural logarithm (ln) of a number (B).

Keystrokes:

- 900 **i** 1 **PV** B **FV** **n** → log B (Base 10)
- x^zy** → ln B (Base e)

Example:

Determine the common logarithm and natural logarithm of 256.

Procedure:

- 900 **i** 1 **PV** 256 **FV** **n** → 2.41
(log, base 10)
- x^zy** → 5.55
(ln, base e)

See Displayed:

LOGARITHMS FOR ANY BASE

The following keystrokes solve for the exponent c in the equation $A^c = B$ when A and B are known. This procedure may be labeled "finding the logarithm of B to the base A".

The natural logarithm of B will again be available in the y register.

Keystrokes:

- A **SAVE** 1 **-** 100 **x** **i** 1 **PV** B **FV** **n** → c
- x^zy** → ln B (base e)

Example:

Find the exponent c in the equation: $16^c = 4096$

Procedure:

- 16 **SAVE** 1 **-** 100 **x** **i** 1 **PV** 4096 **FV** **n** → 3.00
(log 4096, base 16)
- x^zy** → 8.32
(ln 4096, base e)

See Displayed:

Note:

When calculating logarithmic values, features that are required for other purposes are being used. Specifically, when solving for n in the compound interest equation,

$$FV = PV (1 + i/100)^n$$

the HP-80 uses natural logarithms and the expression becomes;

$$n = \frac{\ln (FV/PV)}{\ln (1 + i/100)}$$

Therefore, the keystroke sequence, 900 \boxed{i} 1 \boxed{PV} B \boxed{FV} \boxed{n} , results in the following solution:

$$n = \frac{\ln (B/1)}{\ln 1 + \frac{900}{100}} = \frac{\ln(B)}{\ln (10)} = \log B \text{ (base 10).}$$

Similarly the keystroke sequence,

A $\boxed{SAVE \uparrow}$ 1 $\boxed{-}$ 100 $\boxed{\times}$ \boxed{i} 1 \boxed{PV} B \boxed{FV} \boxed{n} , gives:

$$\frac{\ln B}{\ln A} = \log B \text{ (base A)}$$

As an intermediate step the HP-80 places the $\ln (FV/PV)$ in the y register, and for the values discussed this becomes $\ln (B/1)$ or simply $\ln B$ (base e).

ANTI-LOGARITHMS (BASE e)

The following keystrokes solve for B in the equation $e^c = B$, where e is the base of natural logarithms, and the exponent c is the natural logarithm of B. Step 1 generates the value of e correct to 9 decimal places.

Keystrokes:

1. 1.000001 $\boxed{SAVE \uparrow}$ 1000000 $\boxed{y^x}$ \longrightarrow 2.718281828
2. c $\boxed{y^x}$ \longrightarrow B

Example:

Determine the number whose natural logarithm equals 2.36.

Procedure:

1. 1.000001 $\boxed{SAVE \uparrow}$ 1000000 $\boxed{y^x}$ \longrightarrow 2.72
2. 2.36 $\boxed{y^x}$ \longrightarrow 10.59

See Displayed:

Note:

Since a portion of the value of e repeats itself some will find it easier to remember 2.718281828 than the keystrokes that generate this number. In this case step 1 may be replaced by simply entering e and pressing $\boxed{SAVE \uparrow}$.

ANTI-LOGARITHMS (BASE 10)

The following keystrokes solve for B in the equation $10^c = B$ where the exponent c is the common logarithm of B.

Keystrokes:

- 10 $\boxed{SAVE \uparrow}$ c $\boxed{y^x}$ \longrightarrow B

Example:

Find the number whose common logarithm is 2.41.



Procedure:

10  2.41  

See Displayed:

257.04


“nth” ROOT OF A NUMBER

The key sequence, A  (Gold Key) , where A is a positive number, will calculate the 2nd root (i.e., square root) of A. This operation is commonly written as \sqrt{A} or $\sqrt[2]{A}$. However, the square root of A may also be written in mathematical notation as

$$A^{1/2}$$

and in this form it may be said that A is being raised to the 1/2 power. A more general representation for finding the “nth” root of a number would therefore be:

$$A^{1/n}$$

where n = 2 for square or 2nd root, n = 3 for cube or 3rd root, and so on. Since the  key will raise a positive number to any power, the HP-80 may be used to calculate the “nth” root of positive numbers as shown below.

Keystrokes:

A  1  n    “nth” root of A

Example:

Find the cube (3rd) root of 6859.

Procedure:

6859  1  3   

See Displayed:

19.00

HP-80

APPLICATION NOTES

PUBLISHED AS A SERVICE FOR USERS OF THE HP-80 FINANCIAL POCKET CALCULATOR

JANUARY 17, 1974

NO. 80-008

YIELD AND PERIODIC PAYMENT AMOUNT CALCULATIONS FOR LEASES WITH A BALLOON PAYMENT OR RESIDUAL VALUE

GENERAL

The calculations described in this note assume that the lease payments are *annuity due* (i.e., payments are made at the beginning of the payment periods) and the balloon payment or residual value occurs *at the end of the last payment period*.

The symbolic values listed below will be used to demonstrate the keystroke sequences that follow.

- A = number of payment periods in a year
- B = number of years (and/or fraction of a year as appropriate)
- C = annual yield rate expressed as a percent
- D = periodic payment amount
- E = initial value at the beginning of the lease
- G = balloon payment or residual value amount

ANNUAL YIELD RATE FOR LEASES WITH A BALLOON PAYMENT OR RESIDUAL VALUE

The HP-80 yield to maturity calculation for bonds is used to solve this type of problem. The initial value of the lease corresponds to the bond price. Periodic lease payments correspond to bond coupons, and the balloon payment amount or residual value compares to the redemption (face) value of the bond.

The HP-80 bond calculations have built-in assumptions which are specifically tied to bond calculations.


Some of these are:

- Bond coupons are paid semiannually
- Time is entered in days
- Bond price is expressed as a percent of redemption value

To change these assumptions and use another set of conditions, the data must be adjusted. This is the reason for all the numerical data entries (i.e., 365 \times , 2 \div) and the RCL \div sequence in the general symbolic keystroke solution shown below.

Since the bond algorithm is being used, the same operating limits as expressed in Appendix D of the HP-80 Owner's Handbook apply. For this application these limits can be expressed as follows:

The absolute value of the number entered for PMT must be greater than .125 and less than the value entered for PV . The absolute value of the number entered for PV must be greater than 20 and less than 5000.

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Keystrokes:

1. G **SAVE** D **-** 100 **÷** **STO**
2. A **SAVE** B **×** 2 **÷** 365 **×** **n**
3. D **SAVE** **SAVE** 2 **×** **RCL** **÷** **PMT**
4. **x²y** E **x²y** **-** **RCL** **÷** **PV**
5. **(gold key)** **i** A **×** 2 **÷** → C

Example:

A local truck dealer has offered to lease an off-highway diesel truck for \$80,000.00 (E), requiring equal monthly (A=12) payments of \$1,625.00 (D) for 5 years (B) and a purchase option payment of \$8,000.00 (G) due one month after the last payment. What annual yield will be realized on this contract if the purchase option is exercised?

Procedure:

See Displayed:

- | | |
|---|----------|
| 1. 8000 SAVE 1625 - 100 ÷ STO | 63.75 |
| 2. 12 SAVE 5 × 2 ÷ 365 × n | 10950.00 |
| 3. 1625 SAVE SAVE 2 × RCL ÷ PMT | 50.98 |
| 4. x²y 80000 x²y - RCL ÷ PV | 1229.41 |
| 5. (gold key) i 12 × 2 ÷ | 11.03 |
- (an annual yield rate of 11.03%)

PERIODIC PAYMENT FOR LEASES WITH A BALLOON PAYMENT OR RESIDUAL VALUE

The balloon payment or residual value of a lease in effect reduces the value on which the lease payments are based, thus lowering the payment amount necessary to achieve the desired yield. The following keystrokes find this adjusted initial value and then solve for the periodic payment amount.

Keystrokes:

1. A **SAVE** B **×** **n** C **SAVE** A **÷** **STO** **i**
2. G **FV** **PV**
3. E **x²y** **-** 1 **RCL** **%** **+** **÷**
4. A **SAVE** B **×** **n**
5. **x²y** **RCL** **i** **x²y** **PV**
6. **PMT** → D

Example:

Find the monthly (A=12) payment required for 5 years (B) to achieve an annual yield of 10% (C) if the cost of the equipment is \$63,000.00 (E) and it will have a residual value of \$6,300.00 (G) at the end of the lease.

Procedure:

See Displayed:

- | | |
|--|---------------------------------------|
| 1. 12 SAVE 5 × n 10 SAVE 12 ÷ STO i | 0.83 |
| 2. 6300 FV PV | 3829.07 |
| | (Present value of the residual value) |
| 3. 63000 x²y - 1 RCL % + ÷ | 58681.92 |
| | (Adjusted initial value) |
| 4. 12 SAVE 5 × n | 60.00 |
| 5. x²y RCL i x²y PV | 58681.92 |
| 6. PMT | 1246.82 |
| | (\$1,246.82 monthly payment) |

FEBRUARY 22, 1974

NO. 80-009

INTERNAL RATE OF RETURN (IRR) FOR UNEVEN CASH FLOWS

GENERAL

The *internal rate of return* (sometimes called the *discount rate*, *yield*, or *cost of capital*) is that rate which equates the present value of all future net cash flows to the original investment amount. This is easily found with a simple top row key calculation (n i PMT PV) if the cash flows are equal amounts and equally spaced. An iterative method is required, however, to find the internal rate of return if the cash flows are unequally spaced or are unequal amounts or both. This is accomplished with the HP-80 discounted cash flow analysis procedure shown below.

ITERATIVE METHOD FOR UNEVEN CASH FLOWS

The procedure which follows begins by choosing a "best guess" periodic internal rate of return, and performing the keystrokes for discounted cash flow analysis (see page 37 of the HP-80 Owner's Handbook). If the final net present value is positive, the actual periodic internal rate of return is greater than the rate chosen; if the amount displayed is negative, the actual rate is lower than the rate chosen. Based on the final net present value, different rates can be tried until the final net present value is zero or as close to zero as desired. The internal rate of return will be the last rate used multiplied by the number of cash flow periods per year.

Investments (sometimes called cash outlays) are considered to be negative cash flows, periods with no activity have cash flows of zero, and periods with return or income have positive cash flows. Remember that the HP-80 assumes cash flows occur at the end of the time period.

Keystrokes:

1. \square (gold key) CLX
2. Enter a "best guess" periodic internal rate of return; press i .
3. Enter the amount of the original investment, press CHS to make it negative; press PV .
4. Enter the first period's net cash flow (if it is a cash outlay press CHS); press PV $\Sigma+$ to obtain the current net present value.

Continue step 4 for all subsequent periods.
Zero should be entered for periods with no cash flow.
5. After the last cash flow has been entered, choose a new rate of return based on the net present value shown in the display, and repeat steps 1 through 4. Continue this process until the final net present value is zero or sufficiently close to zero.
6. When the final net present value is zero, the value last entered in step 2 is the periodic internal rate of return. To obtain the annual internal rate of return, multiply this value by the number of cash flow periods per year.

Example:

An investor purchases an apartment complex with a down payment of \$80,000.00. The property is held for 5 years and then sold, resulting in the following net annual cash flows:

-	\$80,000.00	original investment
-	\$13,250.00	end of year 1
	\$14,710.00	end of year 2
	\$16,590.00	end of year 3
	\$22,500.00	end of year 4
	\$109,000.00	end of year 5

What was his internal rate of return for this investment?

Procedure:

See Displayed:

1. (gold key) CLx → 0.00
2. 15 i → 15.00
(a "best guess" of 15% is tried first)
3. 80000 CHS PV → -80000.00
4. 13250 CHS PV Σ+ → -91521.74
- 14710 PV Σ+ → -80398.87
- 16590 PV Σ+ → -69490.67
- 22500 PV Σ+ → -56626.22
- 109000 PV Σ+ → -2433.96
(-\$2,433.96 final net present value)

Since the final net present value is less than zero, a lower rate, 14%, is tried next.

1. (gold key) CLx → 0.00
2. 14 i → 14.00
3. 80000 CHS PV → -80000.00
4. 13250 CHS PV Σ+ → -91622.81
- 14710 PV Σ+ → -80303.94
- 16590 PV Σ+ → -69106.16
- 22500 PV Σ+ → -55784.36
- 109000 PV Σ+ → 826.83
(\$826.83 final net present value)

Since the final net present value is greater than zero, a higher rate, 14.25%, is tried next.

1. (gold key) CLx → 0.00
2. 14.25 i → 14.25
3. 80000 CHS PV → -80000.00
4. 13250 CHS PV Σ+ → -91597.37
- 14710 PV Σ+ → -80327.99
- 16590 PV Σ+ → -69203.56
- 22500 PV Σ+ → -55997.97
- 109000 PV Σ+ → -3.46
(-\$3.46 final net present value)

The internal rate of return for this investment is slightly lower than 14.25%. (It is actually 14.249%)

"BEST GUESS" TECHNIQUES

The preceding example begins with a "best guess" of 15% as the internal rate of return. Any rate could have been chosen to begin with; however, when the "best guess" is close to the actual internal rate of return, fewer iterations are necessary to arrive at a final answer.

There are numerous ways to decide upon a first "best guess". Two methods which accommodate most situations are shown below.

Keystrokes: (One cash flow significantly different from the rest)

If one of the cash flows is significantly different (larger or smaller) from the others, add all the cash flows together and assume that this amount occurs at the time period of this "different" cash flow. A top row key calculation can then be used to solve for a "best guess" periodic internal rate of return.

1. Add together all of the cash flows; press **STO**.
2. Enter the number of periods from the initial investment to the period of the "different" cash flow; press **n**.
3. Enter the amount of the original investment; press **PV**.
4. Press **RCL** **FV** to recall and enter the total of all the cash flows.
5. Press **i** to obtain the value to use for a "best guess" periodic internal rate of return.

Example:

Using the values of the previous example, apply the method shown to calculate a first "best guess" internal rate of return.

Procedure:

- | | | <i>See Displayed:</i> |
|----|--|---|
| 1. | 13250 CHS SAVE 14710 + 16590 + 22500 + 109000 + STO | 149550.00
(\$149,550.00, total of cash flow amounts) |
| 2. | 5 n | 5.00
(period number of the "different" cash flow) |
| 3. | 80000 PV | 80000.00
(original investment amount) |
| 4. | RCL FV | 149550.00 |
| 5. | i | 13.33
(13.33% first "best guess") |

(The actual IRR for this example is 14.249%)

Keystrokes: (No cash flow significantly different from the rest)

If the cash flows are not significantly different from each other, begin by finding their average (arithmetic mean). Then a top row key calculation, which assumes this average cash flow occurs each period, can be used to solve for a first "best guess" periodic internal rate of return.

1. **CLX** (gold key) **CLX**
2. Key in each periodic cash flow amount followed by the **Σ+** key.
3. Press **⌘** to obtain the average cash flow amount; press **STO**.

4. Enter the total number of cash flow periods; press **n** .
5. Press **RCL** **PMT** to recall and enter the average cash flow amount.
6. Enter the amount of the original investment; press **PV** .
7. Press **i** to obtain the value to use for a first “best guess” periodic internal rate of return.

Example:

An original investment of \$100,000.00 resulted in the following annual cash flows:

\$18,000.00	end of year 1
\$24,000.00	end of year 2
\$36,000.00	end of year 3
\$10,000.00	end of year 4
\$41,000.00	end of year 5

Calculate a first “best guess” internal rate of return of this investment.

Procedure:

See Displayed:

1.	[] (gold key) CLx	0.00
2.	18000 Σ+ 24000 Σ+ 36000 Σ+ 10000 Σ+ 41000 Σ+ →	129000.00 (sum of cash flows)
3.	Σ̄ STO →	25800.00 (average periodic cash flow)
4.	5 n →	5.00 (number of cash flow periods)
5.	RCL PMT →	25800.00 (average periodic cash flow)
6.	100000 PV →	100000.00 (original investment)
7.	i →	9.14 (9.14% first “best guess”)

(The actual IRR for this example is 8.3746%)

LINEAR REGRESSION CALCULATIONS

GENERAL

Least squares linear regression is a statistical method for finding a straight line that best fits a set of data points, thus providing a relationship between two variables. The HP-80 Trend Line (**TL**) calculation performs linear regression calculations, but requires that input data be evenly spaced and in chronological order. If data is not evenly spaced, the calculations described below can be used to develop a regression line.

Given the observations of two variables, the HP-80 user can solve for the *slope* (m), and *y-intercept* (b) of the standard regression line equation, $y = mx + b$. From this equation, the dependent variable (y) can be predicted for any given independent variable (x). In addition, the following procedures calculate the *correlation coefficient* (r), which measures the linear relationship between the two variables ($-1 \leq r \leq 1$), the *coefficient of determination* (r^2), which indicates the goodness of fit of the line to the data points ($0 \leq r \leq 1$), and the *standard error* (s) of the estimate of y on x , which is a measure of the scatter about the regression line of y on x .

The following symbolic values will be used to demonstrate the keystroke sequences below.

Input data: $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$

where n = number of observations
 x = independent variable
 y = dependent variable

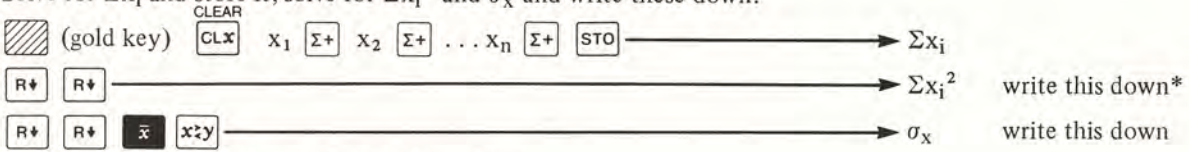
Notation:

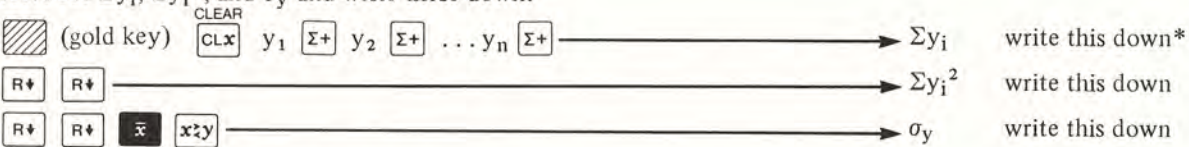
$\sum x_i = x_1 + x_2 + \dots + x_n$	sum of x values
$\sum y_i = y_1 + y_2 + \dots + y_n$	sum of y values
$\sum x_i^2 = x_1^2 + x_2^2 + \dots + x_n^2$	sum of squares of x values
$\sum y_i^2 = y_1^2 + y_2^2 + \dots + y_n^2$	sum of squares of y values
$\sum x_i y_i = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$	sum of $x y$ products
σ_x = standard deviation of x values	
σ_y = standard deviation of y values	
m = slope	
b = y -intercept	
r = correlation coefficient	
r^2 = coefficient of determination	
s = standard error of the estimate of y on x	

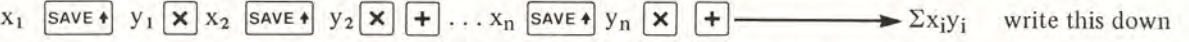
GENERATING TREND LINE AND ESTIMATING VALUES


Keystrokes:


- Solve for Σx_i and store it; solve for Σx_i^2 and σ_x and write these down.



- Solve for Σy_i , Σy_i^2 , and σ_y and write these down.


- Solve for $\Sigma x_i y_i$ and write it down.


- Leaving $\Sigma x_i y_i$ on the display, continue as follows to calculate the slope (m). Write this answer down.


- Calculate, store and write down the y-intercept (b).


- Now the slope (m) and y-intercept (b) can be properly located in the operational stack and storage register so the **TL** key can be used to find the corresponding y value for any given x value (x_k).


- x_k **n** **TL** → y_k
 (This step may be repeated for any x value)

*These intermediate results must be written down in order to complete this set of keystroke procedures; i.e., in order to determine the equation for the regression line. All other results are optional, in that they are only required for computing the correlation coefficient (r), coefficient of determination (r^2), and standard error (s) in the following section.



(NOTE: If a high degree of accuracy is desired, press  (gold key) 6 before writing down intermediate results.)


Example:


A commercial land appraiser has examined 4 vacant lots in the downtown section of a local community, all of which have the same depths but different frontages and values as shown below. Based on this data, what is the relationship between frontage and lot value? What predicted value would a lot have with a 65 foot frontage? With a 50 foot frontage?

Lot Frontage (feet)	Lot Value
70.8	\$10,100.00
60.0	\$ 8,219.00
85.0	\$15,000.00
75.2	\$11,120.00

Procedure:

-  (gold key)  70.8 [Σ+] 60 [Σ+] 85 [Σ+] 75.2 [Σ+] [STO] → 291.00
 (Σx_i , write this down)

 → 21492.68
 (Σx_i^2 , write this down)

 → 10.37
 (σ_x , write this down)

See Displayed:

2. \square (gold key) \square CLEAR \square CL \square x 10100 \square Σ + 8219 \square Σ + 15000 \square Σ + 11120 \square Σ + \longrightarrow 44439.00
 (Σy_i , write this down)
- \square R \square \square R \square \longrightarrow 518216361.0
 (Σy_i^2 , write this down)
- \square R \square \square R \square \square \bar{x} \square \square \bar{y} \square \square \square \longrightarrow 2858.33
 (σ_y , write this down)
3. 70.8 \square SAVE \square 10100 \square \times 60 \square SAVE \square 8219 \square \times \square + 85 \square SAVE \square 15000
 \square \times \square + 75.2 \square SAVE \square 11120 \square \times \square + \longrightarrow 3319444.00
 ($\Sigma x_i y_i$, write this down)
4. \square RCL 44439 \square \times 4 \square \div \square - 21492.68 \square RCL \square SAVE \square \square \times 4 \square \div \square - \square \div \longrightarrow 268.30
 (slope, m, write this down)
5. \square RCL \square \times 44439 \square \square \bar{y} \square - 4 \square \div \longrightarrow -8408.80
 (y-intercept, b, write this down)

(The equation of the regression line is: $y = \$268.30x - \8408.80)

6. 268.30 \square SAVE \square \square SAVE \square \square SAVE \square 8408.80 \square CHS \square STO \longrightarrow -8408.80
7. 65 \square n \square TL \longrightarrow 9030.70
 (\$9,030.70, projected value of lot with 65 foot frontage)
- 50 \square n \square TL \longrightarrow 5006.20
 (\$5,006.20, projected value of lot with 50 foot frontage)

CORRELATION COEFFICIENT AND COEFFICIENT OF DETERMINATION

Keystrokes:

1. Calculate the correlation coefficient (r).
 m \square SAVE \square σ_x \square \times σ_y \square \div \longrightarrow r
2. Leaving the correlation coefficient on the display, find the coefficient of determination (r^2).
 2 \square y^x \longrightarrow r^2

Example:

Using the data from the previous example, what is the correlation coefficient (r) and coefficient of determination (r^2)? The required values written down from that example are:

$$m = 268.30$$

$$\sigma_x = 10.37$$

$$\sigma_y = 2858.33$$

Procedure:

1. 268.30 \square SAVE \square 10.37 \square \times 2858.33 \square \div \longrightarrow .97
 (correlation coefficient, r)
2. 2 \square y^x \longrightarrow .95
 (coefficient of determination, r^2)

See Displayed:

STANDARD ERROR OF THE ESTIMATE OF y ON x

Keystrokes:

- Σy_i^2 [SAVE] b [SAVE] Σy_i [X] [-] m [SAVE]
- $\Sigma x_i y_i$ [X] [-] n [SAVE] 2 [-] \div [gold key] $\sqrt{\frac{x}{y}}$ → s

Example:

Using the data from the first example, what is the standard error(s) of the estimate of y on x?

Given values or values written down from intermediate calculations are:

$$\begin{aligned}\Sigma y_i^2 &= 518216361.0 \\ b &= -8408.80 \\ m &= 268.30 \\ \Sigma y_i &= 44439.00 \\ \Sigma x_i y_i &= 3319444.00 \\ n &= 4\end{aligned}$$

Procedure:

- 518216361 [SAVE] 8408.8 [CHS] [SAVE] 44439 [X] [-] 268.3 [SAVE] → 268.30
- 3319444 [X] [-] 4 [SAVE] 2 [-] \div [gold key] $\sqrt{\frac{x}{y}}$ → 802.56
(standard error, s)

See Displayed:

Equations:

$$m = \frac{\Sigma x_i y_i - \frac{\Sigma x_i \Sigma y_i}{n}}{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}$$

$$b = \frac{1}{n} (\Sigma y_i - m \cdot \Sigma x_i)$$

$$r = \frac{m \sigma_x}{\sigma_y}$$

$$s = \left[\frac{\Sigma y_i^2 - b \Sigma y_i - m \Sigma x_i y_i}{n - 2} \right]^{1/2}$$

$$\sigma_x = \left[\frac{\Sigma (x_i - \bar{x})^2}{n - 1} \right]^{1/2}$$